

F - 1388

**C.B.S. (Third Semester)
EXAMINATION, May - June, 2022**

ESSENTIAL MATHEMATICS

(For Chemistry and Biology Stream)

(CB-301)

Time : Three Hours]

[Maximum Marks:40

Note: Attempt all sections as directed.

(Section-A)

(Objective/Multiple Choice Questions)

(½ marks each)

Note: Attempt all questions-

Choose the correct answer from the options given and writ in your answer book.

1. The Complex analytic function $f(z)$ with real part $e^x \cos y$ is :
- (A) Ze^z
- (B) e^z
- (C) e^{iz}
- (D) None of these

P.T.O.

[2]

2. A real valued function f is harmonic if :

(A) $\partial^2 f / \partial x^2 + \partial f / \partial y = 0$

(B) $\partial^2 f / \partial x + \partial f / \partial y^2 = 0$

(C) $\partial^2 f / \partial x^2 + (\partial f / \partial y)^2 = 0$

(D) None of these

3. The taylor expansion of the function $f(x) = e^{-x}$ is given by:

(A) $e^{-x} = 1 + \frac{x}{1} + \frac{x^2}{2} + \frac{x^3}{3} + \dots$

(B) $e^{-x} = 1 - \frac{x}{1} + \frac{x^2}{2} - \frac{x^3}{3} + \dots$

(C) $e^{-x} = 1 - \frac{x}{1} - \frac{x^2}{2} - \frac{x^3}{3} - \dots$

(D) None of these

4. A solution curve of the equation $x \frac{dy}{dx} = 2y$ passing through (1,2) also passes through.

(A) (2,1)

(B) (0, 0)

(C) (4, 24)

(D) (24, 4)

F - 1388

[3]

5. The homogeneous differential equation $M(x,y) dx + N(x,y) dy = 0$ can be reduced to a differential equations, in which the variables are separated, by the substitution.

(A) $y = vx$

(B) $xy = v$

(C) $x+y = v$

(D) $x-y = v$

6. The differential equation $M(x,y) dx + N(x,y) dy = 0$ is an exact equation if

(A) $\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 0$

(B) $\frac{\partial M}{\partial y} + \frac{\partial N}{\partial x} = 0$

(C) $\frac{\partial N}{\partial y} + \frac{\partial M}{\partial x} = 0$

(D) $\frac{\partial N}{\partial y} - \frac{\partial M}{\partial x} = 0$

[4]

7. If C be an arbitrary constant, the general solution of the equation $(x+2y^3) dx/dy = y$

(A) $x = cy - y^2$

(B) $x = cy - y^3$

(C) $x = cy + y^2$

(D) $x = cy + y^3$

8. The partial differential equation by eliminating the function f from $z = f(y/x)$ is

(A) $x \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$

(B) $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$

(C) $x \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y} = 0$

(D) $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = 0$

[5]

9. Which of the following statement is false-

- (A) $nP_n = x P_n^1 - P_{n-1}^1$
- (B) $(x^2-1)P_n^1 = n x P_n - n P_{n-1}$
- (C) Both (A) & (B)
- (D) None of these

10. Which of the following statement is true-

- (A) $H'_n(x) = 2x H_n(x) - H_{n+1}(x)$
- (B) $H''_n(x) - 2x H'_n(x) + 2nH_n(x) = 0$
- (C) both (A) & (B)
- (D) None

11. Which of the following statement is true-

- (A) $L_n^1(0) = -n$
- (B) $L_n(0) = Ln$
- (C) $L_n(0) = 1$
- (D) All of the above

[6]

12. The Laplace transform of function $t^n e^{-at}$ is -

- (A) $\frac{1}{(s+a)^{n+1}}$
- (B) $\frac{n}{(s+a)^{n+1}}$
- (C) $\frac{n+1}{(s+a)^{n+1}}$
- (D) None of these

13. Laplace transform of $3t^4$ is

- (A) $\frac{18}{s^4}$
- (B) $\frac{24}{s^4}$
- (C) $\frac{72}{s^5}$
- (D) $\frac{12}{s^5}$

[7]

14. Find the sum of the eigen values of the matrix $\begin{bmatrix} 3 & 5 & 7 \\ 4 & 5 & 2 \\ 7 & 9 & 2 \end{bmatrix}$

- (A) 10
- (B) 9
- (C) 8
- (D) 7

15. The eigen value of a 3x3 matrix [A] are given as 2, 3, and 5 then det (A) is

- (A) 10
- (B) 30
- (C) None of these
- (D) Can not be determined

16. The characteristic equation of the matrix $\begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ 2 & 1 & 4 \end{bmatrix}$

- (A) $\lambda^3 + 5\lambda^2 + 2\lambda + 1 = 0$
- (B) $\lambda^3 - 3\lambda^2 + 2\lambda + 1 = 0$
- (C) $\lambda^3 - 5\lambda^2 + 2\lambda - 1 = 0$
- (D) $\lambda^3 - 5\lambda^2 - 2\lambda - 1 = 0$

F - 1388

P.T.O.

[8]

17. Expansion of $\sin^{-1}x$ is

- (A) $x + \frac{x^3}{6} + \frac{3}{40}x^5 + \dots$
- (B) $x - \frac{x^3}{6} + \frac{3}{40}x^5 + \dots$
- (C) $x + \frac{x^3}{6} + \frac{3}{40}x^5 + \dots$
- (D) None of these

18. The harmonic conjugate of function $u = \frac{1}{2} \log(x^2 + y^2)$ is

- (A) $\tan^{-1}(y/x)$
- (B) $\tan^{-1}(x/y)$
- (C) $\cot^{-1}(y/x)$
- (D) None

19. For what value of z , the function $f(z) = \frac{z}{z^2 + 1}$ are continuous.

- (A) continuous every where
- (B) continuous every where except $z = i$
- (C) continuous every where except $z = -i$
- (D) both (B) & (C)

F - 1388

[9]

20. $\int_0^{4+3i} z dz$ along the curve $z = t^2 + it$ is :

- (A) $10 + 8/3 i$
- (B) $30 + 8 i$
- (C) $10/3 + 8 i$
- (D) $10 - 8/3 i$

(Section-B)

(Very Short Answer Type Questions)

(0.75 marks each)

Note: Attempt all questions-

1. Define Euler formula with an example.
2. Define Taylor series.
3. Define partial differential equation.
4. Define order and degree of ordinary differential equation.
5. Define Fourier series.
6. Define Laplace transform.
7. Define eigen value of any matrix.
8. Define characteristics polynomial with an example.
9. Define polar form of complex number.
10. Define harmonic function.

[10]

(Section-C)

(Short Answer Type Questions)

(1.25 marks each)

Note: Attempt all questions-

1. Expand the function $f(x) = \tan x$ by Taylor series expansion method.
2. Expand the function $f(x) = \sin^{-1} x$ by Taylor series expansion method.
3. Solve the differential equation

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 0$$

4. Solve $dy/dx = e^{xy} + x^2e^{-y}$
5. Find the Laplace transform of the function $f(t) = \sin h t$ at
6. Discuss Legendere differential equation with an example
7. Prove that if A is invertible then A^{-1} is invertible and

$$(A^{-1})^{-1} = A$$

8. Find all eigen value and eigen function of matrix $\begin{bmatrix} 1 & 5 \\ 3 & 3 \end{bmatrix}$

9. Discuss cauchy integral formula.
10. Write C-R equation with an example.

[11]

(Section-D)

(Long Answer Type Questions)

(2 marks each)

Note: Attempt all questions-

1. With the help of Taylor series, expand $\sin x$ in term of $(x - \pi/2)$

OR

State and prove Euler theorem.

2. Find the solution of differential equation $\frac{dy}{dx} + 7y = 0$ by series expansion method.

OR

Find a partial differential equation by eliminating a and b from the equation $z = ax + by + a^2 + b^2$.

3. Find the Laplace transform of the following function-

(a) $\cos h$ at

(b) a^t

OR

Write the properties of Hermite polynomials and Lagurre polynomial

F - 1388

P.T.O.

[12]

4. Find all eigen vectors and eign value of the matrix.

$$\begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{bmatrix}$$

OR

Prove that Eigen values of idempotent matrix is only zero and one.

5. Evaluate $\oint_c \frac{e^{zz}}{(z+1)^4} dz$ where c is the circle $|z| = 3$

OR

State & prove Residue theorem.

F - 1388